The Mechanism of Earthquakes According to Dislocation Theory

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The Mechanism of Earthquakes According to Dislocation Theory

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Abstract

The mechanism of earthquakes is presented by use of the physical disolcation theory.

Two assumptions were adopted, i.e. that of shear stress field and that of the field of inhomogeneities in the medium. Space distribution of the stress field and the field of inhomogeneities determine the dynamically active areas. In presence of the stress field the inhomogeneities can be described by loop dislocation.

In this paper the formation of dislocation pairs from loop dislocations with the simultaneous action of the shear field is discussed. The earthquake mechanism is presented as:

(1) A violent movement of the dislocation line or a violent deformation of the dislocation area.

(2) The release of the energy accumulated along the dislocation lines at the moment, when two dislocation areas join or when the dislocation reaches the Earth's su rface.

1 Introduction

In recent times a number of model theories of seismic foci has been formulated. In principle these theories do not deal with the basic nature of the complicated dynamic process at the earthquake focus. They replace the process at the focus by a classical system of equivalent mass forces at a given region and with a given time dependence. The sense of the equivalence of the system of forces with the earthquake process in the focus is based on the approximate equality of the actual displacements of the medium with the displacements calculated for the corresponding system of forces or dynamic model. The equality of the displacements obviously is valid only for distances considerably greater than the size of the focus. In the region of the focus and in its vicinity the deformation law for infinitely small deformations used in model theories does not hold. The classical formulae of LOVE [10] formed the basis of a number of papers dealing with dynamic models of foci. Mention should be made here, above all, of the KEYLIS-BOROK's systematic classification of point foci [11] and a number of papers of this school. An extension of this model was the nonlocal model [16] of foci.

The school of H. HONDA developed along different lines, its model of foci being based on finite dimensions [9].

Expressions for the angular distribution of the energy of seismic volume waves were given on the basis of a model representation of the seismic focus [3]. These expressions contain the immensely important directional character of the seismic processes.

The previously mentioned classical formulae of LOVE for the case of a simple point source are the simplest and most basic element of the theory describing the dynamic model of the focus. They have the following form :

$$u_k^{(1)} = \frac{A_i}{4\pi\rho}\left[\frac{\partial^2}{\partial x_k \partial x_i}\frac{1}{r}\right]\int_{r/a}^{r/c} \xi\, K\,(t-\xi)\, d\xi + \frac{A_k}{4\pi\rho c^2 r}\, K\,(t-\frac{r}{c})$$
$$-\frac{A_i}{4\pi\rho r}\left[\frac{\partial r}{\partial x_k}\right]\left[\frac{\partial r}{\partial x_i}\right]\left[\frac{1}{c^2}\, K\,(t-\frac{r}{c}) - \frac{1}{a^2}\, K\,(t-\frac{r}{a})\right], \qquad (1)$$

where : A_k is the amplitude of the force ; a, c are the velocities of propagation of the seismic P and S waves ; ρ is the density of the medium. The time dependence of the forces is represented by an arbitrary function $K\,(t)$. If the function $K\,(t)$ has a non-continuous character then the value $u_k^{(1)}$ represents the creation of a dislocation encompassing an infinitely small region. Such is the case, for example, if $K\,(t)$ is a step function defined as follows ;

$$K\,(t) = 0 \quad \text{for} \quad t < 0, \qquad K\,(t) = 1 \quad \text{for} \quad t > 0, \qquad (2)$$

where the time $t=0$ denotes the beginning of the earthquake. The dislocation character $u_k^{(1)}$ should be understood in the sense that after integration over a given region of the source the respective values of $u_k^{(1)}$ can give the displacement field for a finite dislocation. In [13], [17] is given a more detailed analysis of this problem. In this analysis the earthquakes, represented by the system of forces with $K(t)$ defined as above, denote the creation of a dislocation at the focus. In the case of a point system of forces such a dislocation embraces an infinitely small region. Model theories understood in this way involve some proposals as regards the nature of the earthquake mechanism. Earthquakes thus involve the appearance of a dislocation element under the influence of stresses prevailing in the medium. The above mechanism of earthquakes does not explain the localization of the focus. The localization is assumed a priori and is not conditioned by the character of the stress field and inhomogeneity of the medium. These theories consider the forces accompanying the earthquake and obviously do not give an explicit relation between the stresses in the medium and the disturbed system of forces at the focus.

The paper of A. VVEDENSKAYA [18] should be included among the papers on this subject. This work, at first glance, considerably departs from the previously discussed theories. A. VVEDENSKAYA based herself directly on dislocation theory. She represents earthquakes, however as the occurrence of a finite dislocation of the disk type. In this sense, therefore, this work belongs to the category of the previously discussed dynamic theories in which the time dependence of the forces is represented by a step function.

In the present paper we should like to give another mechanism of earthquakes basically different from those given thus far. The point of departure for our considerations is the assumption that earthquakes involve two factors. One of them is the stress field in the medium, or, strictly speaking, the nonhydrostatic part of this field. The hydrostatic stresses do not play any role because of their symmetry and of the linearity of the theory. A second basic element is the inhomogeneities appearing in the medium. Inhomogeneities can be generally understood here very broadly; they can be inhomogeneities of the elastic constants, singularities of displacements or physical dislocations. Such inhomogeneities although very small, play the role of attachment points. They constitute a nucleus for release of the stresses appearing in the medium and together with them localize the region of the earthquake focus. Henceforth, as inhomogeneities of the medium of interest to us from the point of view of the theory of the earthquake mechanism we shall take physical dislocations. Physical dislocations constitute that category of inhomogeneities of the medium in which action of the stress field concurs with predisposition of the medium to cracking and sliding in conformity with the field tendencies. Thus, the physical dislocations give a picture of those inhomogeneities of the medium which jointly with the stress field participate in the forming of major dislocation elements. They can therefore not be examined independently from inhomogeneity and stress field. The stress field is connected precisely with singularities of this type.

2 Physical Dislocations

In this section we give the well-known basic ideas of the theory of physcial dislocations [2], [5], [7], [12], [14].

Dislocations represent a specific deformation of an elastic body. The simplest model of a dislocation is a partial cut in one part of the body and the displacement of the material along one side of the cut with respect to the other side. The internal edge of the cut is called a dislocation line (displacements on it are infinite). The main part of the deformation energy is concentrated about it. The essential role in the definition of a physical dislocation is played by the dislocation line. The term "dislocation" is often wrongly used in the meaning of "dislocation line". The displacement of this line causes the extension of the dislocation area. If this line were moved outside the region of the body we obtain a finite displacement of one part of the body with respect to the other without accompanying deformations and stresses. The plane of the dislocation is called the slip plane and the mutual displacement of the material along this plane is given by a dislocation vector called BURGERS' vector [2]. According to whether this vector is perpendicular or parallel to the dislocation line we have to do with dislocations of the edge or screw type. The generalization of the above-described "pure" dislocations is a dislocation with a curvilinear dislocation line, and in a special case a closed curve. This is the so-called dislocation loop. As an elementary dislocation we take a dislocation loop of very small dimensions. The sign of the dislocation is defined by a convention given by BURGERS. The circulation along a dislocation line is connected with a vector normal to the surface of the dislocation in accordance with the right-hand screw law. Crossing the dislocation's area in accordance with the direction of a normal vector, we pass from medium 1 to medium 2. The dislocation defined in this way corresponds to the displacement of material in medium 1 with respect to medium 2 (Fig. 1); [2], [14]. A circulation along the dislocation line is given here by a tangent vector σ_i. We will use the symbol $\perp$ for a positive dislocation and the symbol $\top$ for a negative one.

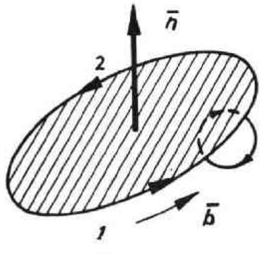

Fig. 1.

If in the medium, in which there is a dislocation, there exists a stress field p_{ik}, then this field interacts with the dislocation through the interaction of dislocation field with the field p_{ik}. This interaction can be described by the action of a force on an element of dislocation line [14].

$$F_k = -\varepsilon_{krs}\, d\sigma_r\, p_{si}\, b_i\,. \tag{3}$$

ε_{krs} equals $+1$ (-1) when k,r,s form an even (odd) permutation of the numbers 1, 2, 3, in the other cases ε_{krs} equals null. The F_k-force is a force of a noncentral type similarly as in the interaction between the magnetic field and a linear current element. Also in close analogy to electrodynamics, is the stress field produced by an element of a dislocation line σ_i.

Under the influence of forces (3) the dislocation can move. The motion takes place either along the slip plane (conservative motion) or in a perpendicular direction (nonconservative motion). The conservative motion corresponds to the deformation tendencies of the dislocation and therefore encounters smaller mechanical resistance. In general the motion of the dislocation can be connected with its deformation e.g. by an increase in the circuit of the loop dislocation. Employing known formulae for the deformation energy we can obtain an expression for the self-energy of the dislocation and the energy of its interaction with an external field. The dislocation energy calculated from these formulae is infinite. In order to avoid this, the dislocation line should be surrounded during integration by a cylinder of finite radius. This is equivalent to taking a dislocation line with a finite thickness. Once more we can, following I.D. ESHELBY [5], point out the analogy to the self-field of an electron.

3 Basic Assumption and Ideas

The basic assumption in the work is that in an elastic medium there exists a field of inhomogeneities. On the basis of our previous considerations we can regard loop dislocations as such elements. The close relation between loop dislocations and inhomogeneities in the medium will be shown at the end of Part 4. For the time being it is sufficient to note that the theory of dislocations is a theory of inhomogeneities in a field of forces. In the case of interest to us this will be the external stress field. Assuming the existence of even small dislocation loops we can show that under the influence of the applied stress field and under the influence of the self-interactions the loops of a given orientation increase in size. Because of this, it is not essential to define the initial size of the dislocation loops. It should be added that the increase in the size of the loops takes place partially at the expense of the energy absorbed from the stress field. Later on, this problem will be discussed in greater detail.

A second basic assumption in this work is the existence of a nonhydrostatic part of the stress field. This field acts on the loop by interaction with the self-field of the dislocation loops. The hydrostatic part of the field is not of basic importance in our considerations and will be neglected hereafter.

We shall consider a shearing stress p_{23} in the medium with one component different from zero. The stress p_{23} interacts with the field of loops with different orientations. Loops of a given orientation are singled out by the shear field, increase in size, and join with similar loops. In this way there arise larger dislocation elements.

For a constant field p_{23} it should be assumed that there exists a fluctuation of the density of the loop distribution and, conversely, for a given loop density there are spatial fluctuations of the field values. We shall see further on that these are in a certain sense and to some extent equivalent. Spatial distribution of loop dislocations and of the stress field determine the areas where dynamic processes start and from which they extend to further parts of the medium.

As the result of the above-described process there arise in the places of maximum density of loops or in the places of maximum of the shear field the greater dislocation elements.

The field p_{23} and the self-field of the dislocation can then cause deformation and motion of the dislocation elements.

4 The Shear Field and Loop Dislocations

The action of the field p_{23} can be represented in the following way. We take loops in the form of small squares with sides given by dx_i. The force acting on one side of the square is given by the relations :

$$dF_k = -\varepsilon_{krs}\, dx_r\, b_i\, p_{si} ,$$

where b_i is the BURGERS' vector of the loop.

It may readily be shown that there act on loops lying in the plane x_1x_3 with BURGERS' vector b_3 the following forces resulting from a fixed stress,

$$dF_3 = -dx_1\, b_3\, p_{23} ,$$
$$dF_1 = dx_3\, b_3\, p_{23} .$$

In the case when loop has the circulation from x_1 to x_3 the forces produce an expansion of the loop (Fig. 2). Loops of the opposite circulation, but with the same BURGERS' vector b_3 shrink owing to the stress field, and in no case do they broaden. If we now examine loops in the plane x_1x_3 with the vecotr b_2 then a couple will act on these loops in such a way as would tend to change their plane. The situation is similar for all loops lying in the plane x_2x_3 and for loops lying in the plane x_1x_2 with the BURGERS' vector b_3. Loops in the plane x_1x_2 with the vector b_2 behave in the field p_{23} in the same way as the previously mentioned loops in the plane x_1x_3 with the vector b_3.

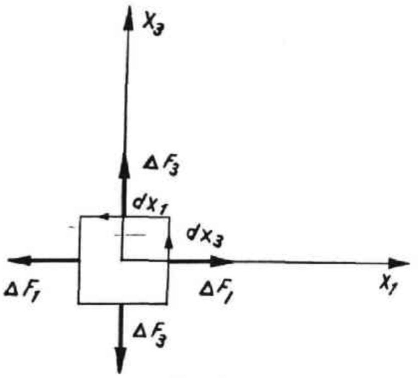

Fig. 2.

It is at once seen that it is sufficient to consider in detail only these two cases, since as the result of the torque produced by the couple acting on the loops of other orientations we can obtain only the same loops in the preferred planes x_1x_3 and x_1x_2; (it is problematical whether such a rotation of the loops is in general possible).

Hence, in planes x_1x_3 and x_1x_2 larger dislocation elements can be formed from loops under the influence of the field p_{23}. As the loops come together and join, one must take into account the interaction of their self-fields; this will be shown below. The situation in both these planes is entirely analogous. It is therefore sufficient to restrict the mathematical considerations to one of them. Additional factors, however, above all, the stress distribution and also greater inhomogeneity density (contact between the two media, and others) can distinguish one of them e.g. x_1x_3 as the principal dislocation plane. Owing to the same factors this plane may be distinguished from among the set of planes prallel to it. This will be the case e.g. if the field is different from zero only in the neighbourhood of this plane. We have, however, to bear in mind that the dislocation processes in a given plane x_1x_3 are usually accompanied by dynamic processes on adjacent elements of planes parallel to x_1x_2.

The action of the field on the loops as a whole can be expressed by calculating the total force acting on the loops. With this aim in mind we integrate (along the loop) the elements of force acting on elements of the loop circuit,

$$F_k = -\oint \varepsilon_{krs}\, b_i\, p_{si}\, dx_r\,.$$

Using STOKES' theorem we obtain

$$F_k = \iint \varepsilon_{rnm}\, \varepsilon_{rks}\, b_i \frac{\partial p_{si}}{\partial x_m}\, n_n\, dS\,,$$

where the integration runs over the surface of the loop ΔS. Owing to the small dimensions of the loop we have

$$F_k = \left(n_k\, b_i \frac{\partial p_{mi}}{\partial x_m} - n_s\, b_i \frac{\partial p_{si}}{\partial x_k}\right) \Delta S\,. \tag{4}$$

For loops in the plane x_1x_3 with the vector b_3 we find

$$F_1 = -b_3 \frac{\partial p_{23}}{\partial x_1} \Delta S\,,$$

$$F_2 = b_3 \left(\frac{\partial p_{31}}{\partial x_1} + \frac{\partial p_{33}}{\partial x_3}\right) \Delta S\,, \tag{4a}$$

$$F_3 = -b_3 \frac{\partial p_{23}}{\partial x_3} \Delta S .$$

From these formulae it follows that the constant external field p_{23} does not act on the loops as a whole, but act only on its elements (expanding or contracting the loops).

We continue to assume that the field to be constant, and limit ourselves to the consideration of the interaction of loops of the same orientation.

For loops in the plane x_1x_3 with the vector b_3 we have, according to BURGERS the following expressions for the displacement [2], [13]:

$$\begin{aligned} u_1 &= -\frac{c^2}{4\pi}\left[\frac{3x_1x_2x_3}{r^5}\left(\frac{1}{c^2}-\frac{1}{a^2}\right)\right] b_3\, \Delta S , \\ u_2 &= -\frac{c^2}{4\pi}\left[\frac{3x_2^2x_3}{r^5}\left(\frac{1}{c^2}-\frac{1}{a^2}\right)+\frac{x_3}{a^2 r^3}\right] b_3\, \Delta S , \\ u_3 &= -\frac{c^2}{4\pi}\left[\frac{3x_2x_3^2}{r^5}\left(\frac{1}{c^2}-\frac{1}{a^2}\right)-\frac{x_2}{a^2 r^3}\right] b_3\, \Delta S . \end{aligned} \tag{5}$$

From these formulae we obtain the stress field produced by the loop:

$$\begin{aligned} p_{11} &= -\frac{c^2}{4\pi}\left[\lambda\, \frac{6x_2x_3}{a^2r^5}+2\mu\, \left(-\frac{3x_2x_3}{r^5}+\frac{15x_1^2x_2x_3}{r^7}\right)\left(\frac{1}{c^2}-\frac{1}{a^2}\right)\right] b_3\, \Delta S , \\ p_{22} &= -\frac{c^2}{4\pi}\left[\lambda\, \frac{6x_2x_3}{a^2r^5}+2\mu\left\{\left(-\frac{6x_2x_3}{r^5}+\frac{15x_2^3x_3}{r^7}\right)\left(\frac{1}{c^2}-\frac{1}{a^2}\right)+\frac{3x_2x_3}{a^2r^5}\right\}\right] b_3\, \Delta S , \\ p_{33} &= -\frac{c^2}{4\pi}\left[\lambda\, \frac{6x_2x_3}{a^2r^5}+2\mu\left\{\left(-\frac{6x_2x_3}{r^5}+\frac{15x_2x_3^2}{r^7}\right)\left(\frac{1}{c^2}-\frac{1}{a^2}\right)+\frac{3x_2x_3}{a^2r^5}\right\}\right] b_3\, \Delta S , \\ p_{12} &= -\frac{c^2\mu}{4\pi}\left[\left(\frac{1}{c^2}-\frac{1}{a^2}\right)\left(-\frac{3x_1x_3}{r^5}+\frac{30x_1x_2^2x_3}{r^7}\right)+\frac{3x_1x_3}{a^2r^5}\right] b_3\, \Delta S , \\ p_{13} &= -\frac{c^2\mu}{4\pi}\left[\left(\frac{1}{c^2}-\frac{1}{a^2}\right)\left(-\frac{3x_1x_2}{r^5}+\frac{30x_1x_2x_3^2}{r^7}\right)+\frac{3x_2x_3}{a^2r^5}\right] b_3\, \Delta S , \\ p_{23} &= -\frac{c^2\mu}{4\pi}\left[\left(\frac{1}{c^2}-\frac{1}{a^2}\right)\left\{-\frac{3\,(x_2^2+x_3^2)}{r^5}+\frac{30x_2^2x_3^2}{r^7}\right\}+\frac{3\,(x_2^2+x_3^2)}{a^2r^5}-\frac{2}{a^2r^3}\right] b_3\, \Delta S . \end{aligned} \tag{6}$$

Inserting (6) into (4a) we obtain expressions for the force between the loops defined by the normal vector and the BURGERS' vector b_3. To facilitate the calculations we shall give this interaction for three of the simplest loop positions, which obviously does not limit the generality of the treatment.

(1) The loops lie on the x_1 axis. The absolute value of the force between them is expressed as follows ($F_2 = F_3 = 0$):

$$\left|F_1\right| = \frac{3\mu}{2\pi}\, \frac{c^2b_3^2\Delta S^2}{a^2r^4} \tag{7}$$

where this force is attractive.

(2) The loops are on the x_3 axis. The absolute value of the force is given by ($F_1 = F_2 = 0$)

$$\left|F_3\right| = \frac{3\mu c^2}{4\pi r^4}\left|\frac{3}{c^2}-\frac{4}{a^2}\right| b_3^2 \Delta S^2. \tag{8}$$

For $\frac{a^2}{c^2} > \frac{4}{3}$ the force F_3 is attractive. The given inequality is always satisfied for values of a, c encountered in the Earth.

(3) The loops lie on the x_2 axis, and therefore outside the plane x_1x_3 which we are considering. The only component of the force F_2 different from zero acts on the loop "pulling" in into the plane.

From the above considerations it follows that the self-interaction between loops causes them to concentrate in the plane under consideration. As a result of the attraction between the loops, the loops can join to form larger dislocation elements. Thus the interaction is superimposed on the applied field p_{23}. The joining of the loops in the plane x_1x_3 causes an increase in the surface enclosed by the dislocation in the direction x_1 or x_3 (cases 1 and 2). On the other hand the joining of the loops in parallel planes (case 3) causes the BURGERS' vector to increase several times in magnitude, but with the dislocation enclosing the same area. It is now seen from the above considerations that the external field p_{23} and the self-field of the loops interact with one another, as has been already pointed out. We recall that the increase in the loops (i.e. loops with preferred orientations) takes place at the expense of the energy of the external field.

We have thus shown that for a constant field the concentration of the dislocations takes place as a result of the interaction between the loops. As has already been noted, this is possible only with fluctuations of the loop density. On the other hand, with a fixed density, the concentration can be the result of a spatial change in external field p_{23}. This question has a much more profound aspect, since the exact duality of the problem can be shown. We assume e.g. a homogeneous distribution of the loop density on the plane x_1x_3; then using formulae (6) it is found that the resultant stress field is given by

$$p_{23} = \frac{nb_3c^2}{4\rho_0}\,\mu\left(\frac{3}{c^2} - \frac{2}{a^2}\right), \tag{9}$$

where : n is the number of microloops on a unit area; ρ_0 is the loop radius. Here all other components of the tensor p_{ik} are equal to zero. In formula (9) we can pass to the limit $n \to \infty$, $\rho_0 \to 0$, $b_3 \to 0$ with the condition that $\lim \frac{nb_3}{\rho_0} = \text{const.}$ Generalizing, we can say that the existence inside the medium of certain stresses is caused by some microloop density distribution and vice versa. This is a particular type of mechanism for transferring the stresses in the medium. This view, on the other hand, is confirmed by the consideration of the meaning of dislocations in the presence of internal stresses, as occurs for example, in the process of hardening.

On the basis of (9) it can be shown that in the field p_{23} the dislocation loops of larger size are formed around the inhomogeneities of the medium. In this way dislocation loops can represent inhomogeneities and we can somewhat modify our assumption in Part 3. We assume now that in the presence of shear field there arise the loop dislocations of a given orientation. The occurrence of loop dislocations can be established particularly clearly in the case of incompressible intrusions where the formation of dislocations compensates the absence of stresses in the region of the intrusion.

5 Dislocation Pairs and Their Deformation

Employing the results of Part 4, we can express the results as follows. Under the influence of the field p_{23} and the interaction of the loops, the loops concentrate and build up in the preferred plane x_1x_3 (and also in adjacent elements of the planes parallel to x_1x_2). As a result, there arise larger dislocation elements in the directions x_1 and x_3. We shall consider two extreme cases.

(1) As a result of the grouping and building up of the loops along the axis x_1 there arise two linear dislocations of opposite sign (dislocation pair of the edge type).

(2) The grouping and building up of dislocations along the x_3 axis give a pair of linear dislocations of the screw type.

The possibility of formation of pairs of dislocations from loops was presented by NABARRO [13] who treated the problem from the mathematical point of view. The aim of NABARRO's paper was to determine, on the basis of the pairs formation, the field of moving dislocations. We can now give the physical meaning of NABARRO's synthesis of loops.

Let us now assume that the distributions of the fluctuations of the loop density (or the stress distribution of the field p_{23}) is such that on our plane x_1x_3 their dislocation elemenets form primarily along the x_1 axis. In the limiting case we shall therefore be dealing with pairs of edge dislocation. In the more general case this will be an edge pair (a, b) of finite length l_0 bounded by sides of the screw type (c, d) of length R (Fig. 3). We shall now consider the displacement field of the pairs at

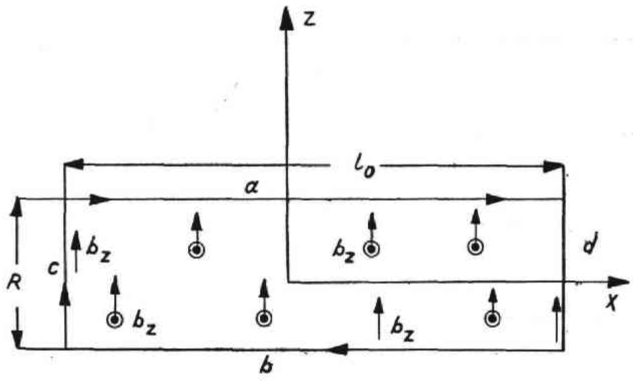

Fig. 3.

formation. Under the influence of the field p_{23} (and the interaction of other dislocations under formation) the individual sides of the dislocation move away from one another and form separate linear dislocations of opposite sign. For $l_0 \gg R$ we can, in approximation, assume that we are dealing with two linear edge dislocations. Such a representation facilitates the calculations. In a more detailed analysis, however, it should be remembered that we have finite dislocations and that the edge dislocations are in this case bounded on the sides by two screw dislocations. The above picture should be supplemented by the interaction of the linear dislocations. The force acting on an element of unit length of the edge dislocation and derived from the second edge dislocation (primed) is [6]:

$$F_3 = -b_3 p_{23}' , \tag{10}$$

where

$$p_{23}' = \frac{\mu b_3'}{2\pi(1-\sigma)} \frac{1}{x_3' - x_3} ,$$

x_3 is the coordinate of the first dislocation on the plane x_1x_3; x_3' refers to the second dislocation; similarly b_3, b_3' are the corresponding BURGERS' vectors; σ is the Poisson's ratio. From the above formula it follows that two dislocations of opposite sign (see the convention on the sign and direction around the loop in Part 2) attract each other and those with the same signs repel each other.

If as a result of the synthesis of the loops a pair of edge dislocations forms at a distance R from each other, then between them there is an attractive force ($b_3 = b_3'$)

$$F_3 = \frac{\mu b^2_3}{2\pi(1-\sigma) R} . \tag{11}$$

The field p_{23} interacting with the dislocation in accordance with (3) must exceed the magnitude of the force given above in order that the dislocations move apart. Our considerations apply only for large l_0, as had been assumed.

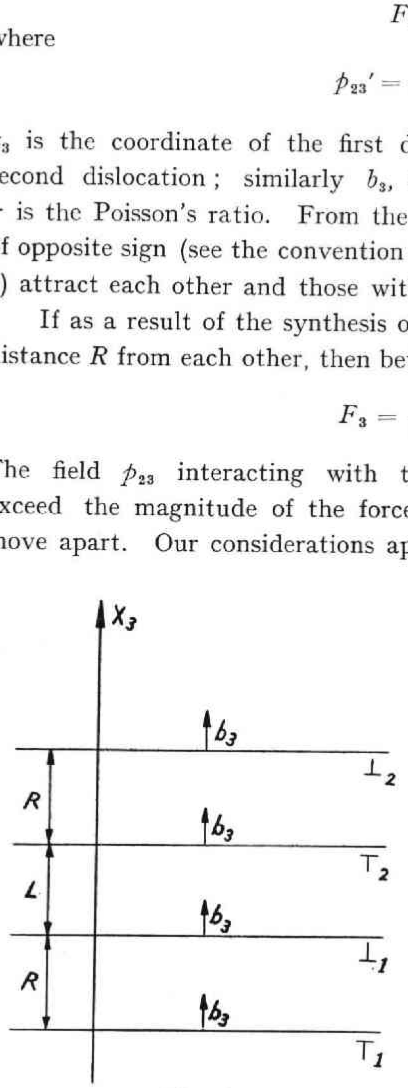

Fig. 4.

For small l_0 we must return to the mechanism of the formation of finite pairs of dislocations from the loops. If the initial loops have a radius ρ_0 then the pair being formed will be built up from this value in the direction of the axes x_1, and x_3. The initial conditions for such a building up is $p_{23}^{\text{exter}} > p_{23}^{\text{loop}}$ (cf. [6]).

Let us now return to the situation for separate dislocation. Besides the field p_{23}, the field of other dislocations can influence the separation of the dislocations. If on plane x_1x_3 two systems of dislocation pairs form, obviously of the same orientation (since we are dealing the action of the same external field p_{23}), then the internal dislocations $\perp_1, \top_2$ will attract themselves (Fig. 4), the resulting expression for the force will in this case have the form

$$\left|F_3\right| = p_{23} b_3 + b_3^2 \frac{\mu}{2\pi(1-\sigma)} \left(\frac{1}{L} - \frac{1}{R} - \frac{1}{L+R}\right) ,$$

where L is the distance of the internal dislocations $\perp_1$, $\top_2$. The external dislocations $\perp_2$ and $\top_1$ will for a sufficiently strong stress field travel in the external directions from centres O, O′ and the resultant force will be expressed by the relation

$$\left|F_3\right| = p_{23} b_3 + b_3^2 \frac{\mu}{2\pi(1-\sigma)} \left(\frac{1}{L+R} - \frac{1}{R} - \frac{1}{L+2R}\right),$$

where the friction force accompanying the motion of the dislocations has been neglected. This expression defines the conditions of motion and equilibrium of the dislocations. In addition, if the dislocation is at a distance H from the Earth's surface then to the previous expressions there should be added the interaction of the free surface. This interaction can be expressed by means of the well-known image method. Here the image is a dislocation of the opposite sign. Such a resultant field gives vanishing stresses on the Earth's surface. In this way the interaction of the free surface can be expressed as an interaction between the dislocation and its image. This interaction is always expressed as the attraction of the dislocation in the direction of the surface. In this case one should therefore add to the previous expressions the term

$$F_3 = b_3^2 \frac{\mu}{2\pi(1-\sigma)} \frac{1}{2H}, \tag{12}$$

which, of course, primarily concerns the dislocation $\perp_2$.

We supplement the above treatment by giving expressions for the displacements accompanying the individual dislocation process. If at the time $t=0$ a pair of edge dislocations is formed violently by the mechanism of grouping and loop growth, then the field of its displacements u_k^c is expressed by the formulae [17],

$$\begin{aligned}
u_1^c &= 0,\\
u_2^c &= x_3 b_3 R \frac{c^2}{\pi a^2}\left(-\frac{x_2^2 at}{r^4 A} + \frac{Aat}{r^4} - \frac{4x_2^2 Aat}{r^6}\right) && \left(\text{for } \frac{r}{a} \leq t\right),\\
&\quad - x_3 b_3 R \frac{1}{\pi}\left(-\frac{x_2^2 ct}{r^4 C} + \frac{Cct}{r^4} - \frac{4x_2^2 Cct}{r^6} + \frac{1}{2}\frac{ct}{r^2 C}\right) && \left(\text{for } \frac{r}{c} \leq t\right),\\
u_3^c &= x_2 b_3 R \frac{c^2}{\pi a^2}\left(-\frac{x_3^2 at}{r^4 A} + \frac{Aat}{r^4} - \frac{4x_3^2 Aat}{r^6}\right) && \left(\text{for } \frac{r}{a} \leq t\right), \quad (13)\\
&\quad - x_2 b_3 R \frac{1}{\pi}\left(-\frac{x_3^2 ct}{r^4 C} + \frac{Cct}{r^4} - \frac{4x_3^2 Cct}{r^6} + \frac{1}{2}\frac{ct}{r^2 C}\right) && \left(\text{for } \frac{r}{c} \leq t\right),
\end{aligned}$$

where: $A = \sqrt{a^2 t^2 - r^2}$, $C = \sqrt{c^2 t^2 - r^2}$; $r^2 = x_2^2 + x_3^2$.

R is the distance between the dislocations of the pair under creation.

In the case of the gradual formation of an edge pair through successive stages of pairs of finite length, we find that the field of the initial displacements in the perpendicular plane will be equal to the field of the initial displacements of a suddenly formed infinite dislocations.

Here one can also cite the results of A. VVEDENSKAYA [18] representing the field of initial displacements of suddenly formed finite dislocation of the disk type. In what follows it will be convenient to work with pairs, which, in effect, lead to linear dislocations.

The field of a moving single linear edge dislocation is expressed as follows (case where the dislocation has a large velocity v) [17]:

$$u_1 = 0\,,$$

$$u_2 = \frac{b}{2\pi}\,\frac{x_2{}^2 c^2}{\bar{r}^2}\left[\frac{\left(1-\frac{v^2}{c^2}\right)^{3/2}}{c^2} - \frac{\left(1-\frac{v^2}{a^2}\right)^{3/2}}{a^2}\right] + \frac{b}{4\pi}\,\frac{v^2}{c^2}\left(1-\frac{v^2}{a^2}\right)^{-1/2}\ln\left\{x_2{}^2\left(1-\frac{v^2}{c^2}\right)+\bar{x}_3{}^2\right\},$$

$$u_3 = \frac{b}{2\pi}\,\frac{x_2\bar{x}_3 c^2}{\bar{r}^2}\left[\frac{\left(1-\frac{v^2}{c^2}\right)^{3/2}}{c^2} - \frac{\left(1-\frac{v^2}{a^2}\right)^{3/2}}{a^2}\right] \tag{14}$$

$$-\frac{b}{2\pi}\,\frac{x_2}{\bar{x}_3}\left[\left(1-\frac{v^2}{c^2}\right)^{1/2} - \frac{2v^2\left(1-\frac{v^2}{c^2}\right)^{1/2}}{c^2} - \frac{2v^2c^2\left(1-\frac{v^2}{c^2}\right)^{1/2}}{a^4}\right],$$

where:

$$x_3 = \bar{x}_3 - vt\,,$$

$$\bar{r}^2 = x_2{}^2 + \bar{x}_3{}^2\,.$$

We can also give proper expressions when the dislocations meet. If they meet at the time $t=0$ then the displacements corresponding to their annihilation will be given by the formulae

$$u_k^a = u_k^s - u_k^c \tag{15}$$

where: u_k^c is given by (13), R is replaced by L; L is the distance between the dislocations before their annihilation, u_k^s is the displacement field of the pair of dislocation in the static case [13];

$$u_1{}^s = 0\,,$$

$$u_2{}^s = b_3 L\,\frac{1}{\pi}\left(1-\frac{c^2}{a^2}\right)\frac{x_2{}^2 x_3}{r^4} + b_3 L\,\frac{1}{2\pi}\,\frac{c^2}{a^2}\,\frac{x_3}{r^2}\,,$$

$$u_3{}^s = b_3 L\,\frac{1}{\pi}\left(1-\frac{c^2}{a^2}\right)\frac{x_2 x_3{}^2}{r^4} + b_4 L\,\frac{1}{2\pi}\,\frac{c^2}{a^2}\,\frac{x_2}{r^2}\,.$$

A similar consideration can be made in the case when junction of the loop dislocations takes place in the direction of axis x_3. As the result we obtain pairs of screw dislocation characterized by the fact that the dislocation vector is parallel to the dislocation line. Expressions analogous to expressions (13), (14), (15) for dislocations of the screw type are given in the paper of NABARRO [13].

6 Earthquake Mechanism

On the basis of the above considerations we can give a hypothesis for the earthquake mechanism. In particular, as causes for earthquakes we shall consider:

(1) Violent formation of larger dislocation elements (we include in this type

violent motions of dislocation lines and deformation of dislocations)-this corresponds, in principle to mechanism of H. HONDA, KEYLIS-BOROK, VVEDENSKAYA, and others, but according to our interpretation the chief part of the stress energy is transformed here into the increase of the internal energy of the dislocation. This will be typical of weaker earthquakes.

(2) Release of the dislocation energy at the border of the medium, or upon the meeting of two dislocation lines of opposite sign (annihilation)--we shall regard this mechanism as typical of stronger earthquakes. These processes take place violently --this is caused by the increase in the forces as the dislocation lines approach each other or approach the surface of the Earth.

In the case of earthquakes of type 1 the displacements that occur are specified by the formulae describing the creation, expansion, motion, etc. of the dislocations.

In this kind of earthquake the work of the external field p_{23} goes primarily into the increase of the potential energy of the dislocation and only a part of it appears in the form of seismic waves. When, however, a single dislocation line moves steadily, the whole work of the p_{23} field is converted into deformation energy without any loss of seismic radiation.

Hereafter we shall consider the case 2 which describes the basic mechanism of earthquakes; earthquakes of type 1, however, can be regarded as precursor shocks. In order to describe the energy release of the dislocation we shall examine the scheme in Fig. 5. We assume that on the plane x_1x_3 two pairs of edge dislocations $\perp_1$, $\top_1$ and $\perp_2$, $\top_2$ form around the generating centres O and O′. In this case of the resultant forces acting in the directions denoted by arrows--this will always be the case for a suitably large p_{23}--we have the situation shown in Fig. 5.

In region "b" dislocations $\perp_1$, $\top_2$ approach and annihilate each other; this process being accompanied by a release of their energy. When the two unlike dislocations approach each other, the deformation takes place at the expence of the energy of stress field as well as the interaction energy of the dislocations. We assume that the dislocations initially approach each other very slowly and then, when at a distance L, there occurs violent motion and the dislocations join. The seismic radiation begins at the moment when the accelerated movements begin. Their potential energy at a distance L is (KOEHLER [12], COTTRELL [21]) as follows.

Edge dislocations :
$$E = \frac{\mu}{2\pi(1-\sigma)}\left\{\ln\frac{L}{r_0} - \frac{1}{2}\right\} l_0 b_3^2 \ . \tag{16}$$

Screw dislocations :
$$E = \frac{\mu}{2\pi} b^2 l_0 \ln\frac{L}{r_0} \ , \tag{16a}$$

where : r_0 is the radius of the dislocation line--a small quantity; l_0 is the length of the dislocation. The above expression represents the self-energy and the interaction energy of the dislocation $\perp_1$, $\top_2$. As a result of the violent motion this energy is partially changed into kinetic energy, then the total energy is released on joining. Some part of this energy is converted into the work of deformation on joining. In this manner we can adopt the formulae given by KOEHLER and COTTRELL for the case of energy release in focus. The work of the field p_{23} and of the resistance forces was not taken into account. When the edge dislocations are under consideration, the displacement field during annihilation is given by the formula (15). The annihilation of the dislocations $\perp_1, \top_2$ corresponds to the joining of dislocation regions enclosed by lines $\perp_1, \top_1$ and $\perp_2$, $\top_2$ in one region of the deformation. This region further increases as a result of the motion of the external dislocation $\perp_2$ towards the boundary of the medium, i.e. towards the surface of the Earth. On the surface of the Earth there takes place a release of the energy of dislocation $\perp_2$ which is connected with the arrival of the deformation at the surface (region "a"). The expected deformation of the Earth's surface is shown by a dotted line (Figs. 5, 6) when the edge dislocation is under consideration. In the case of screw dislocation the surface deformation should be of the same type, but lying in horizontal plane.

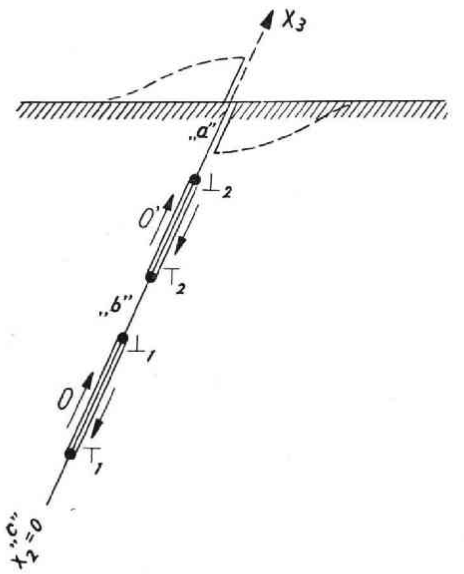

Fig. 5.

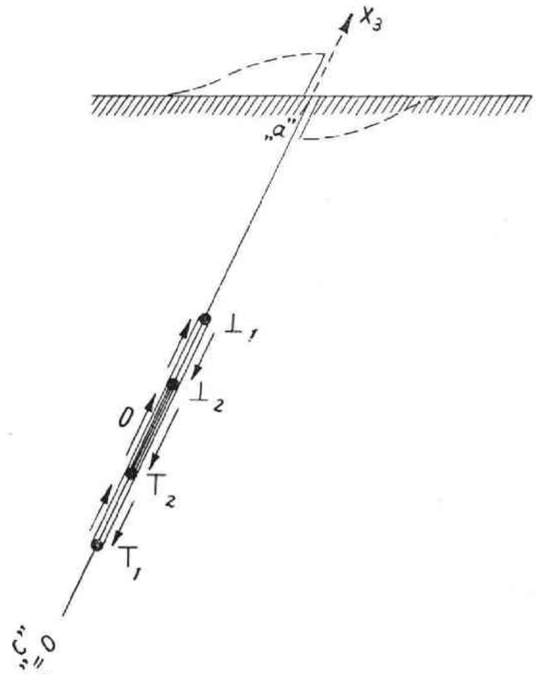

Fig. 6.

If one takes into account the fact that the stress field is expressed in the case under consideration by the sum of the dislocation field and its image, one may readily show that the energy releases will be given by a formula analogous to (16) with a factor 1/2.

Edge dislocation (displacement in vertical plane) :

$$E = \frac{1}{2} \frac{\mu}{2\pi(1-\sigma)} \left\{ \ln \frac{H}{r_0} - \frac{1}{2} \right\} l_0 b_3^2 . \tag{17}$$

Screw dislocation (displacement in horizontal plane) :

$$E = \frac{1}{2} \frac{\mu}{2\pi} l_0 b_3^2 \ln \frac{H}{r_0} , \tag{17a}$$

where : H is the distance of the dislocation from the surface of the Earth at the instant the dislocation begins to move violently towards the boundary of the medium i.e. at the moment when its rapid motion towards the Earth's surface is to take place : H may be called the depth of the first impulse. The H-quantity range may be taken at the interval of some tens of kilometers. The radius of the dislocation line r_0 determines the radius of area in which occur the extreme stresses. The elastic theory should hold outside of r_0 which implies for r_0 a quantity several times greater than the dislocation vector b_3 [21]. Now we can tentatively assume that $\ln \frac{H}{r_0} \approx 10$.

It follows from formulae (16) and (17) that for the same l_0 and b_3 the energy of the surface earthquakes ("a") will be one-half the energy of the earthquakes of mean depth ("b").

In region "c" the dislocation $\top_1$ moves deeper into the Earth producing a deformation of the medium in this direction. In this way the region of the dislocation will increase in this direction too. It should be assumed that in the deeper layers there also occurs a release of the stresses represented by the dislocation $\top_1$. The character of this release, however, may be more gradual than in the previously discussed cases, but at this time it is difficult to give a precise description of it.

Let us now consider, for simplicity, the case in which only one generating centre of a physical dislocation exists on the dislocation plane (Fig. 6). We assume that it is concentrated around the line "O". As a result of the constant action of the field p_{23} multipole pairs of dislocations form around "O". The individual dislocation pairs move away from one another, as was described above. As a result there arises the situation shown in Fig. 6. In this case the neighbouring dislocations $\perp_1$, and $\perp_2$ are of the same sign and act on each other repulsively. The resulting process is as if the external dislocation lines are repulsed in the external directions by dislocations lying close to the centre "O".

As a consequence of the action of the field p_{23} and the interactions of the dislocations being formed, the externaly situated dislocation $\perp_1$ moves to the surface of the Earth where it releases its energy. The resulting force acting on the next dislocation line is changed and gives now the opportunity for a quick approach of this line to the surface. In consequence, its energy will be released and so a replica takes place. Finally, in this case the dynamic processes will be analogous to those discussed previously in region "a" and also in "c". The general case will be a combination of the above-discussed special cases.

We supplement the above considerations by data concerning the energy release of a physical dislocation on the surface of the Earth with the process accompanying the motion of the deformation towards the surface. In Table 1, is given a comparison of the energies calculated from (17), (17a) with the energies calculated from the magnitudes in accordance with the formula given by GUTENBERG-RICHTER [8]

$$\log E = 11 + 1.6\, M \,. \tag{18}$$

We should, however, bear in mind that these formulae represent the total energy released in the focus. One part of this energy works along the sides of the slip (fault), the other is emitted as seismic radiation. The above calculation was made for earthquakes for which the length of the dislocation and value of the slip, were known. From (17) and (18) it can be concluded that the magnitude of the earthquakes can not greatly exceed the value 8. Otherwise the size of the dislocation would be comparable with the size of the Earth itself.

Table 1.

Earthquake [15], [19], [20]	Slip Length (km)	Slip Amplitude (m)	Energy according to (17) and (17a) (ergs)	Earthquake Magnitude (M)	Energy according to (18) (ergs)
California, 1906	450	3.05	1.2×10^{24}	$8^1/_4$	1.6×10^{24}
Nevada, 1915	30	5	1×10^{24}	$7^3/_4$	2.1×10^{23}
Mino-Owari, 1891	65-120	7	3.5×10^{24}	$7^1/_2$	1.0×10^{23}
Assam, 1897	20	12	2.5×10^{24}	$8^1/_2$	4.0×10^{24}
Nevada, 1954	80	1.85	8.2×10^{22}	$7^1/_4$	4.0×10^{22}
Tango, 1927	30	1.5	2×10^{22}	7.4	6.9×10^{22}
North Izu, 1930	15	2.5	1.8×10^{22}	7	1.7×10^{22}

7 Conclusions

Our basic assumption was that there exists in the medium a stress field p_{23} and a given distribution of elementary loop dislocations representing inhomogeneities in the stress field. The field p_{23} causes them to group and build up. As a result, larger dislocation elements form. Assuming that the maximal values of the shear field concentrate in the vicinity of the plane x_1x_3, we restrict ourselves to the examination of dislocation processes on this plane (the hypocentral plane). A necessary condition for the development of these processes is the nonuniform distribution of the field p_{23} or the loop density. Here there exists a particular kind of duality of the concepts of field and loops. Since the field p_{23} acts in the vicinity of the plane $x_2=0$ it also acts on elements of the plane x_3=const. on which there may arise similar processes, but to a lesser extent. It thus follows that along the principal dislocation plane smaller dislocations form on the plane perpendicular to it.

The dislocation element presented by pairs of dislocation lines limiting the dislocation area is typical and simple. There may occur edge as well as screw dislocation pairs. Under the influence of the shear field the dislocation area spreads which can be expressed by movements of the dislocation limiting lines.

The violent increase of the dislocation elements or the violent motion of the dislocation lines can be regarded as an earthquake. Earthquakes of this type correspond, in principle, to the mechanism given by VVEDENSKAYA [18]. The difference here is that VVEDENSKAYA regards earthquakes as the occurrence of dislocations, while in our approach this effect is understood in the sense of a violent expansion up of the dislocations from the elementary loops.

As discussed at the beginning, the mechanism given by KEYLIS-BOROK, H. HONDA and others refers also to the occurrence of elementary dislocations.

We believe that the basic mechanism of earthquakes depends on the release of the energy amassed around the dislocation lines, either as a result of the annihilation of two dislocation lines of opposite sign (junction of two dislocation area) or as a release at the boundary of the medium when the dislocation reaches the Earth's surface.

For earthquakes of this type formulae for the displacement and energy release have been given.

The earthquake theory presented here is also a theory of geological dislocations and of the processes connected with them.

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References

1. BILBY, B.A., *Proc. Phys. Soc.* A, **63**, 191, 1950.
2. BURGERS, J.M., *Proc. Acad. Sci. Amst.* **42**, 293, 1939.
3. DROSTE, S., *Acta Geoph. Pol.* **4**, 215, 1957.
4. ESHELBY, I. D., *Proc. Roy. Soc. A*, **197**, 396, 1949.
5. ESHELBY, I.D., FRANK, F.C., NABARRO, F.R.N., *Phil. Mag.* **41**, 351, 1951.
6. ESHELBY, I.D., *Phil. Trans. Roy. Soc. Lond. A.* **244**, No. 877, 87, 1951.
7. FRANK, F.C., *Proc. Phys. Soc.* A, **62**, 131, 1949.
8. GUTENBERG, B., RICHTER, C.F., *Seismicity of the Earth*, Princeton Univ. Press., 1949.
9. HONDA, H., *Sci. Rep. Tohoku. Univ. Ser*, **5**, *Geophys.* **9**, Supp. 1957.
10. LOVE, A., *A Treatise on the Mathematical Theory of Elasticity*, New York, 1944.
11. KEYLIS-BOROK, W.I., D.A.N., USSR, **70**, nr. 6, 1959.
12. KOEHLER, I.G., *Phys. Rev.* **60**, 397, 1941.
13. NABARRO, F.R.N., *Phil. Mag.* **122**, 1224, 1951.
14. PEACH, M., I.C. KOEHLER, *Phys. Rev.* **80**, 436, 1950.
15. SAVARENSKI, E.F.,KIRNOS, D.P., *Elements of Seismology and Seismometry*, Moscow 1955.
16. TEISSEYRE, R., *Acta Geoph. Pol.* **4**, 226, 1957.
17. TEISSEYRE, R., Problems of Synthesis and Motion of Dislocations, (in preparation).
18. VVEDENSKAYA, A. V., *Izw A. USSR*, No. **3**, 277. 1956.
19. BEYRLY, P., DENOYER J., *Contributions in Geophysics: In Honor of Beno Gutenberg*, p. 17–35, Pergamon Press., 1958.
20. KASAHARA K., *Bull. Earthquake Research Inst.* **35**, 473, 1957.
21. COTTRELL, A.H., *Dislocation and Plastic Flow in Crystals*, Oxford, 1956.